

# Intratabilidade e Otimização

Luciana Buriol	Instituto de Informática, UFRGS
Eduardo Uchoa	Departamento de Engenharia de Produção, UFF
Celina Figueiredo	Engenharia de Sistemas e Computação, UFRJ

encontro de teoria da computação  
congresso da sbc · porto alegre · 2016



David Johnson  
1945–2016

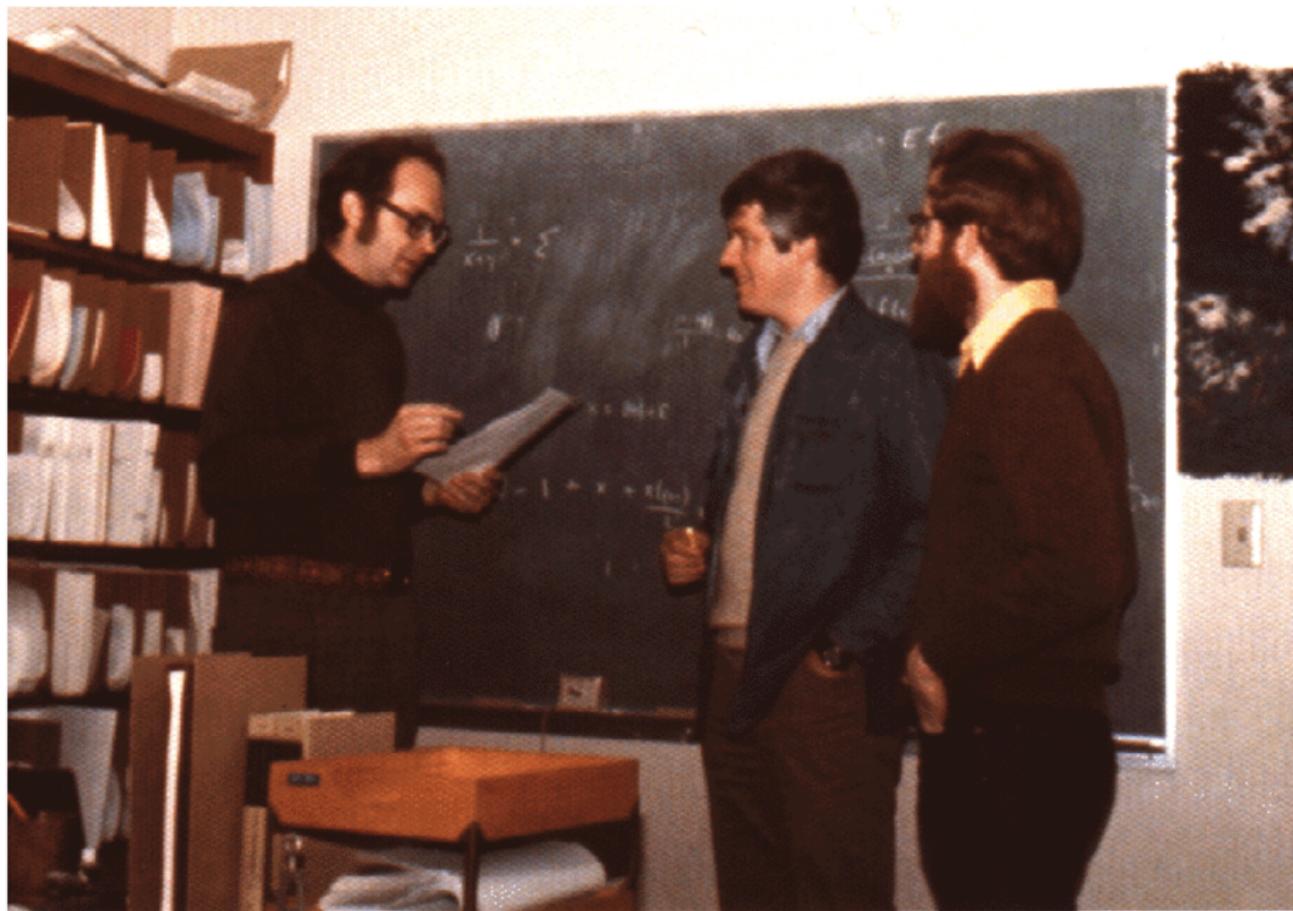
## Bio – AT&T Labs

- ▶ Americano, matemático, pesquisador  
área de Ciência da Computação, Algoritmos e Otimização
- ▶ Mestrado (1968) e Doutorado (1973) no MIT  
“Near-Optimal Bin Packing Algorithms”
- ▶ 1973–2013: AT&T Bell Labs / AT&T Labs Research:
  - ▶ 1988: chefe do depto de Fundamentos Matemáticos de Computação
  - ▶ 1996: chefe do depto de Algoritmos e Otimização
  - ▶ 2013: membro honorário
- ▶ 2014–2016: Columbia University

## Bio – AT&T Labs – Down the Hall

- ▶ M.R. Garey, R. L. Graham, D.S. Johnson, and D.E. Knuth  
Complexity results for bandwidth minimization  
SIAM J. Appl. Math. 34 (1978), 477–495
  
- ▶ M.R. Garey, D.S. Johnson, and R.E. Tarjan  
The planar Hamiltonian circuit problem is NP-complete  
SIAM J. Computing 5 (1976), 704–714

# Knuth – Garey – Johnson



# Tarjan – Garey – Johnson



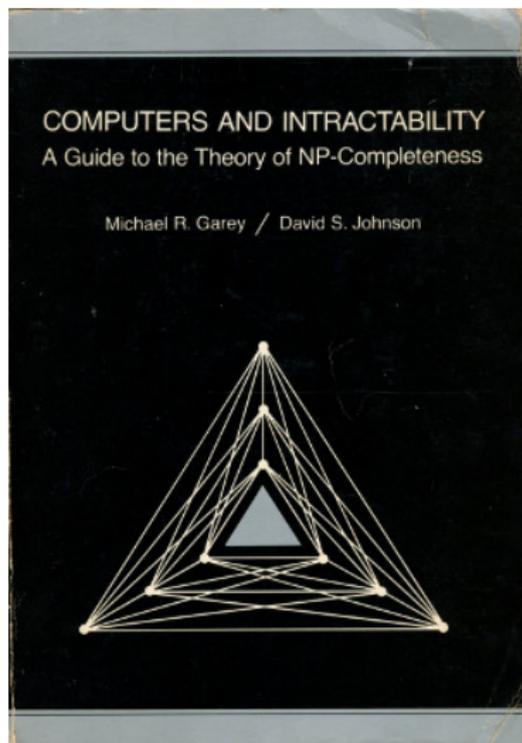
## Bio – Service: Book, Editor, Chair

- ▶ 1979: “Computers and Intractability: A Guide to the Theory of NP-Completeness” (livro com Michael Garey)
- ▶ 1982: An Ongoing Guide on NP-completeness  
Journal of Algorithms / ACM Transactions on Algorithms
- ▶ 1990: fundou o ACM-SIAM Symposium on Discrete Algorithms (SODA), foi committee chair 25 anos
- ▶ 1987–1991: chair do ACM Special Interest Group on Algorithms and Computation Theory (SIGACT)
- ▶ 1983–1987: editor do Journal of the ACM
- ▶ 2004–2016: associate editor of ACM Transactions on Algorithms (TALG)

## Bio – Prizes

- ▶ 1995 ACM Fellow  
contribuições fundamentais para as teorias de algoritmos aproximativos e de complexidade computacional, e para ACM
- ▶ 1997 SIGACT Distinguished Service Prize  
dedicação generosa e iniciativa pessoal para Teoria da Computação
- ▶ 2010 Knuth Prize, ACM SIGACT  
contribuições para teoria e análise de algoritmos, algoritmos para problemas de otimização, para encontrar a melhor solução dentre as soluções viáveis, para a teoria dos problemas NP-completos, para identificar os problemas difíceis de resolver eficientemente, para técnicas de aproximação e a definição de soluções quase ótimas, contribuições para a análise teórica e experimental de algoritmos
- ▶ 2016 Member of National Academy of Engineering

# The Guide – Computers and Intractability



“Despite that 23 years have passed since its publication, I consider Garey and Johnson the single most important book on my office bookshelf. Every computer scientist should have this book on their shelves as well. NP-completeness is the single most important concept to come out of theoretical computer science and no book covers it as well as Garey and Johnson.”

Lance Fortnow, “Great Books: Computers and Intractability: A Guide to the Theory of NP-Completeness”

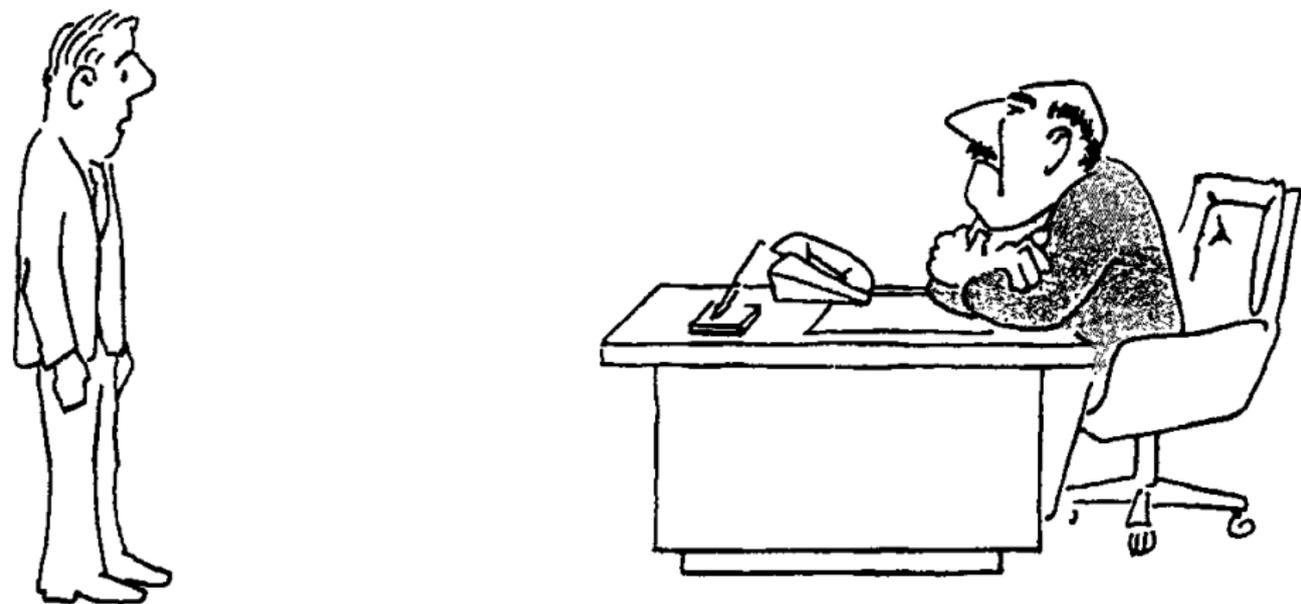
NP-completo: simboliza o abismo da intratabilidade inerente para resolver problemas maiores e mais complexos

Variedade ampla de problemas frequentes: matemática, computação, pesquisa operacional

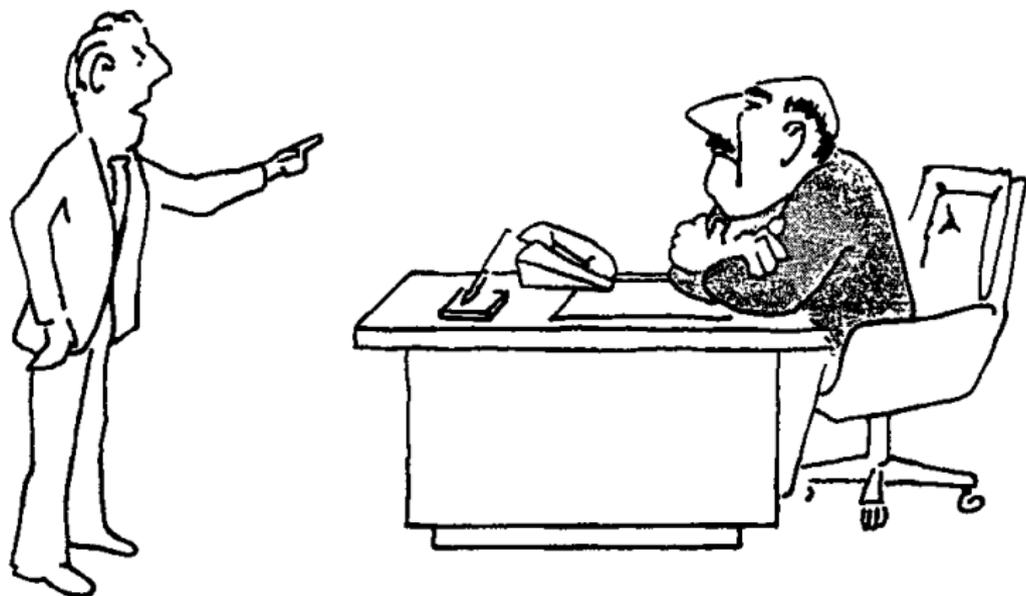
- ▶ Capítulos 1–5: teoria básica
- ▶ Capítulos 6–7: aproximação, hierarquia de classes de complexidade
- ▶ Apêndice: metade do livro! Lista bem organizada de problemas



“Bandersnatches are the subject of a difficult algorithm design project for an apparently NP-complete problem.”



“I can’t find an **efficient** algorithm, I guess I’m just too dumb.”



**“I can’t find an efficient algorithm, because no such algorithm is possible!”**



“I can’t find an efficient algorithm, but neither can all these famous people.”

# The Lost Cartoon



WE MAY NOT BE ABLE TO SOLVE IT...  
BUT WE SURE CAN GET CLOSE !

- ▶ Graph isomorphism
- ▶ Subgraph homeomorphism (for a fixed graph  $H$ )
- ▶ Graph genus
- ▶ Chordal graph completion
- ▶ Chromatic index
- ▶ Spanning tree parity problem
- ▶ Partial order dimension
- ▶ Precedence constrained 3-processor scheduling
- ▶ Linear programming
- ▶ Total unimodularity
- ▶ Composite number
- ▶ Minimum length triangulation

# Ongoing Guide – Os 12 problemas atualizados em 2005

Problem Name	Source	Status	Covered in
GRAPH ISOMORPHISM	[G&J]	Open	–
SUBGRAPH HOMEOMORPHISM (FOR A FIXED GRAPH H)	[G&J]	P	[Col 19, 1987]
GRAPH GENUS	[G&J]	NPC	[Col 21, 1988]
CHORDAL GRAPH COMPLETION	[G&J]	NPC	[Col 1, 1981]
CHROMATIC INDEX	[G&J]	NPC	[Col 1, 1981]
PARTIAL ORDER DIMENSION	[G&J]	NPC	[Col 1, 1981]
PRECEDENCE CONSTRAINED 3-PROCESSOR SCHEDULING	[G&J]	Open	–
LINEAR PROGRAMMING	[G&J]	P	[Col 1, 1981]
TOTAL UNIMODULARITY	[G&J]	P	[Col 1, 1981]
SPANNING TREE PARITY PROBLEM	[G&J]	P	[Col 1, 1981]
COMPOSITE NUMBER	[G&J]	P	This Column
MINIMUM LENGTH TRIANGULATION	[G&J]	Open	–
IMPERFECT GRAPH	[Col 1, 1981]	P	This Column
GRAPH THICKNESS	[Col 2, 1982]	NPC	[Col 5, 1982]
EVEN COVER (MINIMUM WEIGHT CODEWORD)	[Col 3, 1982]	NPC	This Column
“UNRESTRICTED” TWO-LAYER CHANNEL ROUTING	[Col 5, 1982]	Open	–
GRACEFUL GRAPH	[Col 6, 1983]	Open	–
ANDREEV’S PROBLEM	[Col 17, 1986]	Open	–
SHORTEST VECTOR IN A LATTICE	[Col 18, 1986]	“NPC”	This Column

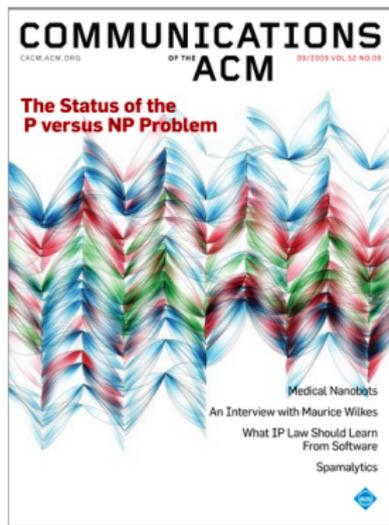
# Ongoing Guide – Graph Restrictions and Their Effect

GRAPH CLASS	MEMBER	INDSET	CLIQUE	CLIPAR	CHRNUM	CHRIND	HAMCIR	DOMSET	MAXCUT	STTREE	GRAISO
Trees/Forests	P [T]	P [GJ]	P [GJ]	P [T]	P [GJ]						
Almost Trees ( $k$ )	P	P [24]	P [T]	P?	P?	P?	P?	P [45]	P?	P?	P?
Partial $k$ -Trees	P [2]	P [1]	P [T]	P?	P [1]	O?	P [3]	P [3]	P?	P?	O?
Bandwidth- $k$	P [68]	P [64]	P [T]	P?	P [64]	P?	P?	P [64]	P [64]	P?	P [58]
Degree- $k$	P [T]	N [GJ]	P [T]	N [GJ]	N [GJ]	N [49]	N [GJ]	N [GJ]	N [GJ]	N [GJ]	P [58]
Planar	P [GJ]	N [GJ]	P [T]	N [10]	N [GJ]	O	N [GJ]	N [GJ]	P [GJ]	N [35]	P [GJ]
Series Parallel	P [79]	P [75]	P [T]	P?	P [74]	P [74]	P [74]	P [54]	P [GJ]	P [82]	P [GJ]
Outerplanar	P	P [6]	P [T]	P [6]	P [67]	P [67]	P [T]	P [6]	P [GJ]	P [81]	P [GJ]
Halin	P	P [6]	P [T]	P [6]	P [74]	P [74]	P [T]	P [6]	P [GJ]	P?	P [GJ]
$k$ -Outerplanar	P	P [6]	P [T]	P [6]	P [6]	O?	P [6]	P [6]	P [GJ]	P?	P [GJ]
Grid	P	P [GJ]	P [T]	P [GJ]	P [T]	P [GJ]	N [51]	N [55]	P [T]	N [35]	P [GJ]
$K_{3,3}$ -Free	P [4]	N [GJ]	P [T]	N [10]	N [GJ]	O?	N [GJ]	N [GJ]	P [5]	N [GJ]	O?
Thickness- $k$	N [60]	N [GJ]	P [T]	N [10]	N [GJ]	N [49]	N [GJ]	N [GJ]	N [7]	N [GJ]	O?
Genus- $k$	P [34]	N [GJ]	P [T]	N [10]	N [GJ]	O?	N [GJ]	N [GJ]	O?	N [GJ]	P [61]
Perfect	O!	P [42]	P [42]	P [42]	P [42]	O?	N [1]	N [14]	O?	N [GJ]	I [GJ]
Chordal	P [76]	P [40]	P [40]	P [40]	P [40]	O?	N [22]	N [14]	O?	N [83]	I [GJ]
Split	P [40]	O?	N [22]	N [19]	O?	N [83]	I [15]				
Strongly Chordal	P [31]	P [40]	P [40]	P [40]	P [40]	O?	O?	P [32]	O?	P [83]	O?
Comparability	P [40]	O?	N [1]	N [28]	O?	N [GJ]	I [GJ]				
Bipartite	P [T]	P [GJ]	P [T]	P [GJ]	P [T]	P [GJ]	N [1]	N [28]	P [T]	N [GJ]	I [GJ]
Permutation	P [40]	O?	O	P [33]	O?	P [23]	P [21]				
Cographs	P [T]	P [40]	P [40]	P [40]	P [40]	O?	P [25]	P [33]	O?	P [23]	P [25]
Undirected Path	P [39]	P [40]	P [40]	P [40]	P [40]	O?	O?	N [16]	O?	O?	I [GJ]
Directed Path	P [38]	P [40]	P [40]	P [40]	P [40]	O?	O?	P [16]	O?	P [83]	O?
Interval	P [17]	P [44]	P [44]	P [44]	P [44]	O?	P [53]	P [16]	O?	P [83]	P [57]
Circular Arc	P [78]	P [44]	P [50]	P [44]	N [36]	O?	O?	P [13]	O?	P [83]	O?
Circle	P [71]	P [GJ]	P [50]	O?	N [36]	O?	P [12]	O?	O?	P [70]	O?
Proper Circ. Arc	P [77]	P [44]	P [50]	P [44]	P [66]	O?	P [12]	P [13]	O?	P [83]	O?
Edge (or Line)	P [47]	P [GJ]	P [T]	N [GJ]	N [49]	O?	N [11]	N [GJ]	O?	N [70]	I [15]
Claw-Free	P [T]	P [63]	O?	N [GJ]	N [49]	O?	N [11]	N [GJ]	O?	N [70]	I [15]

# Perspectivas – O Problema do Milênio

Problema central em Teoria da Computação: P versus NP

Existe pergunta cuja resposta pode ser verificada rapidamente, mas cuja resposta requer muito tempo para ser encontrada?



setembro 2009

- ▶ Real world instances were not as worst-case or asymptotic as our theory is.
- ▶ Champion algorithms from the theory world could be outclassed by ad hoc algorithms with much worse (or unknown) worst-case behavior.
- ▶ Some algorithms and ideas from the theory world have been successfully applied, often to purposes for which they were not originally designed.
- ▶ Algorithms from the Operations Research and Metaheuristic communities have perhaps had more real-world impact on coping with NP-hardness than those from TCS.